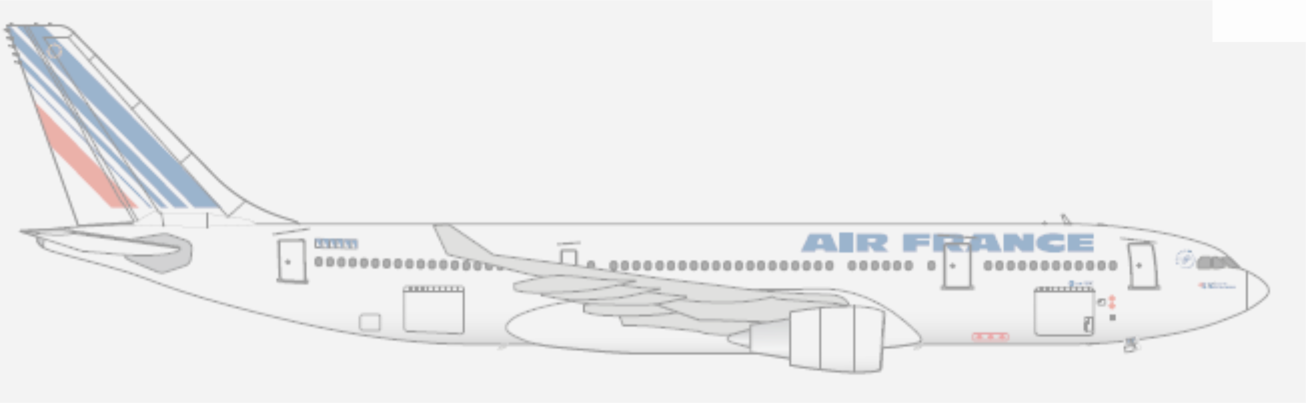

Bayesian Search for Missing Aircraft

Fusion 2015 Plenary Talk
8 July 2015

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Bayesian search theory provides a principled and successful recipe for planning searches for lost aircraft and other objects

About Me

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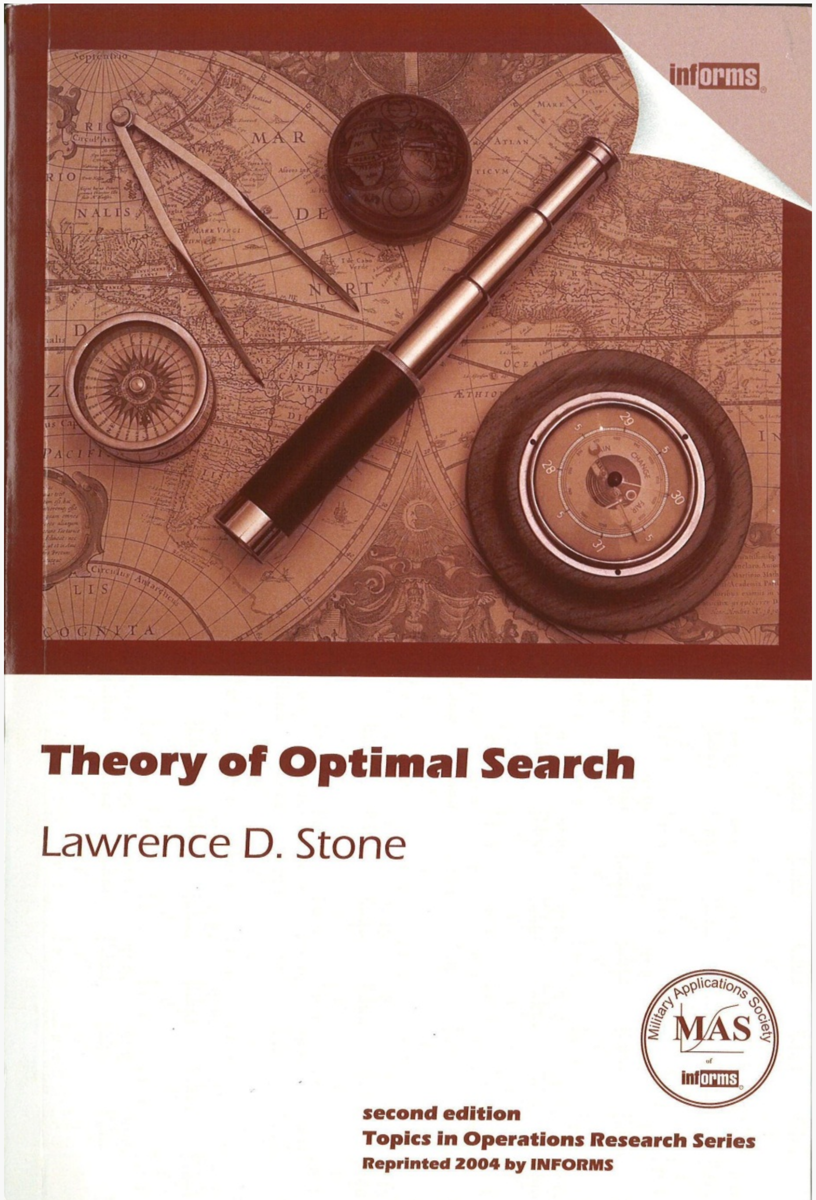
Outline

- Prior successes applying Bayesian search techniques
- What do we mean by Bayesian search?
- The search for Air France flight 447
- How would we apply Bayesian search planning to Malaysia Airlines MH370?

Metron Search Background



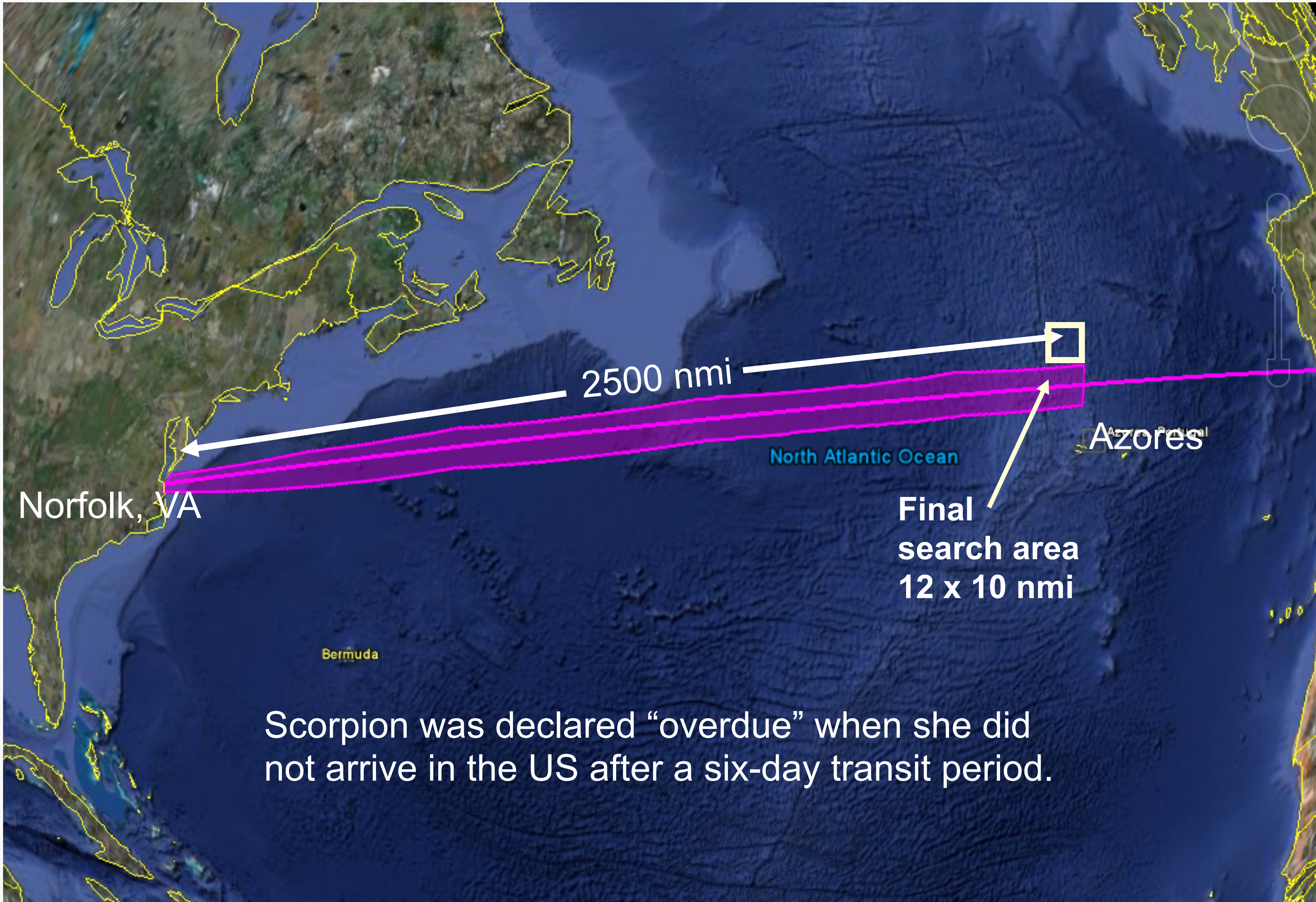
- Dr. Larry Stone's Lanchester Prize Book:

  Theory of Optimal Search, 1975
- Searches:
  - US Submarine *Scorpion,* 1968
  - *SS Central America,* 1988
  - AF 447, 2011
- Search Planning Aids: US Coast Guard
  - CASP, 1974

    Computer Assisted SAR Planning
  - SAROPS, 2007

    Search & Rescue Optimal Planning System

USS Scorpion Initial Uncertainty Area



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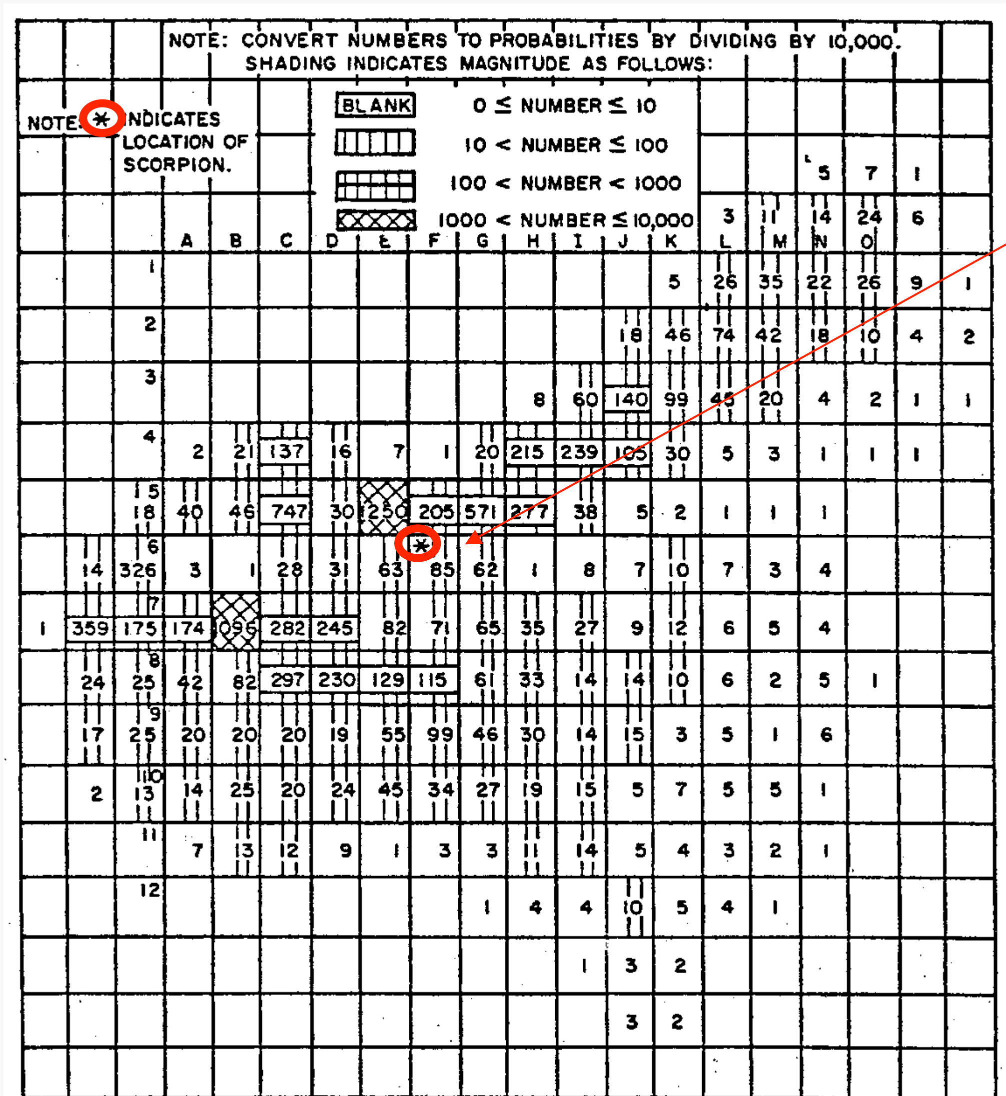
Probability Map for Wreck Location



Scorpion located within 260 yards of highest probability cell

SS *Central America*

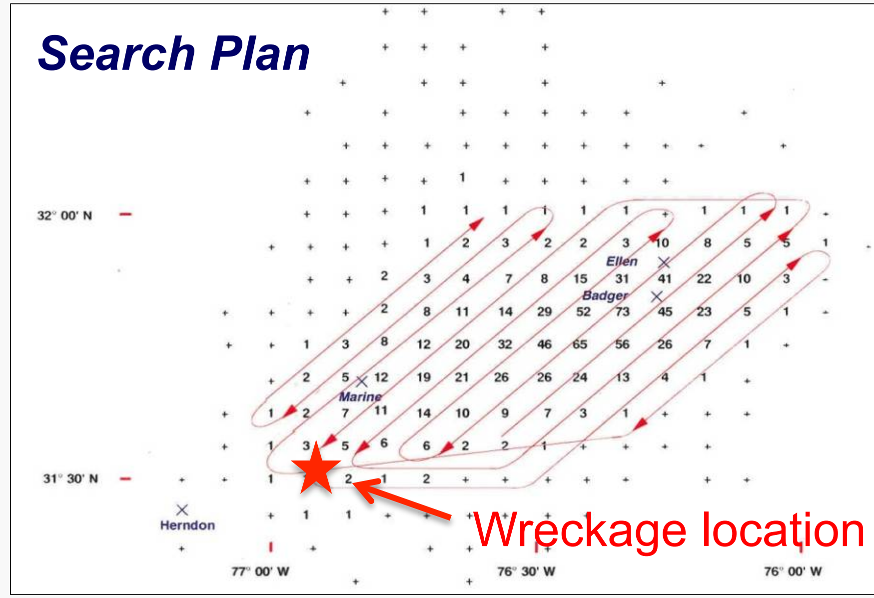
- In 1857, transiting from California to New York, the *SS Central America* sank in a September hurricane taking tons of gold bars and coins to the ocean bottom 8,000 feet below.

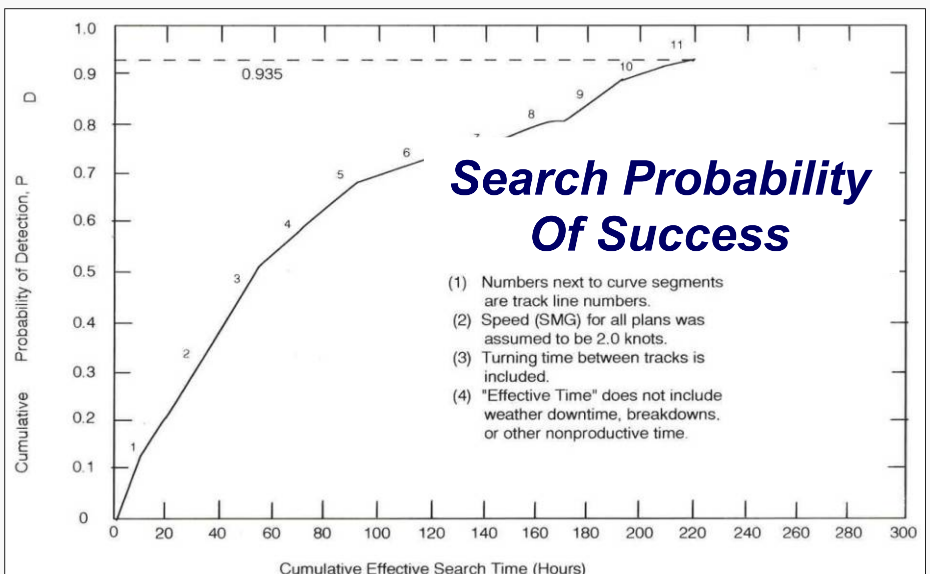
- Some 425 people lost their lives, including the Captain of the ship.
- A financial panic ensued in New York, and the Navy was directed to investigate the loss.

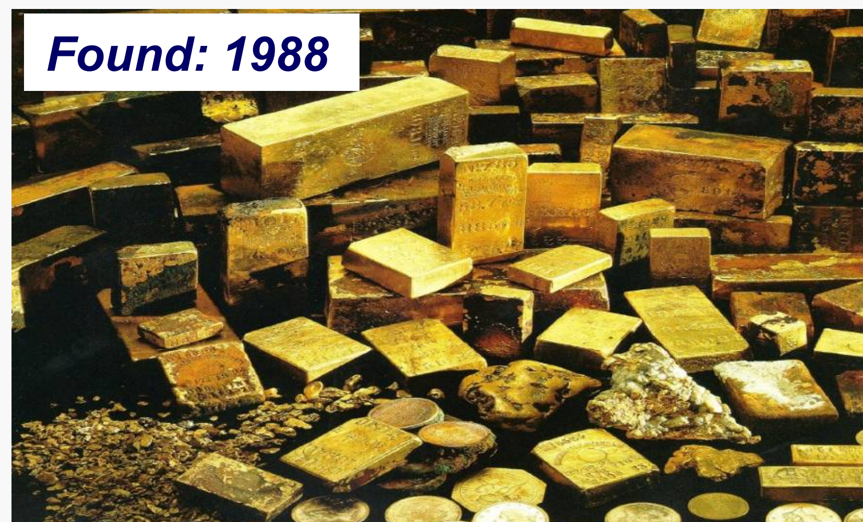
- The *Central America* was the most famous shipwreck of its time, comparable to the loss of the *Titanic* in the 20th Century.

- In 1985 The Columbus Discovery Group hired Dr. Larry Stone to produce a probabilty map to guide search efforts.

Search for the *S.S. Central America*







Bayes Rule:

A simple but rigorous approach to updating beliefs as new information becomes available.

Bayesian Search Planning Process - 1

Step 1: Construct Prior Distribution

- Organize information into consistent subsets = Scenarios
- Quantify uncertainties in terms of probabilities
  - Common to work with incomplete data
  - Each scenario results in a probability distribution for the target location
  - Include probabilities that scenarios are correct – often subjective.
- Combine into a weighted Prior : weights = scenario probabilities
- Resulting distribution represents the decision maker's best understanding of the problem
  - Basis for search planning decisions

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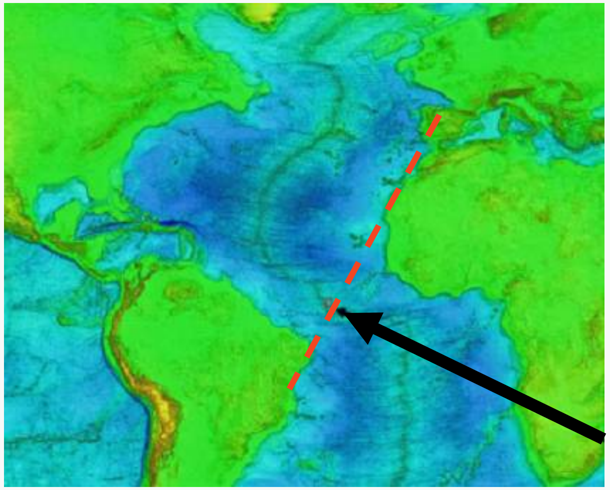
Bayesian Search Planning Process - 2

Step 2: Go out and Search

- Allocate search effort to maximize detection probability
- If search fails, compute Posterior given failure
- Use Posterior to plan next increment of search

Air France Flight 447: Open Ocean Search for Wreckage

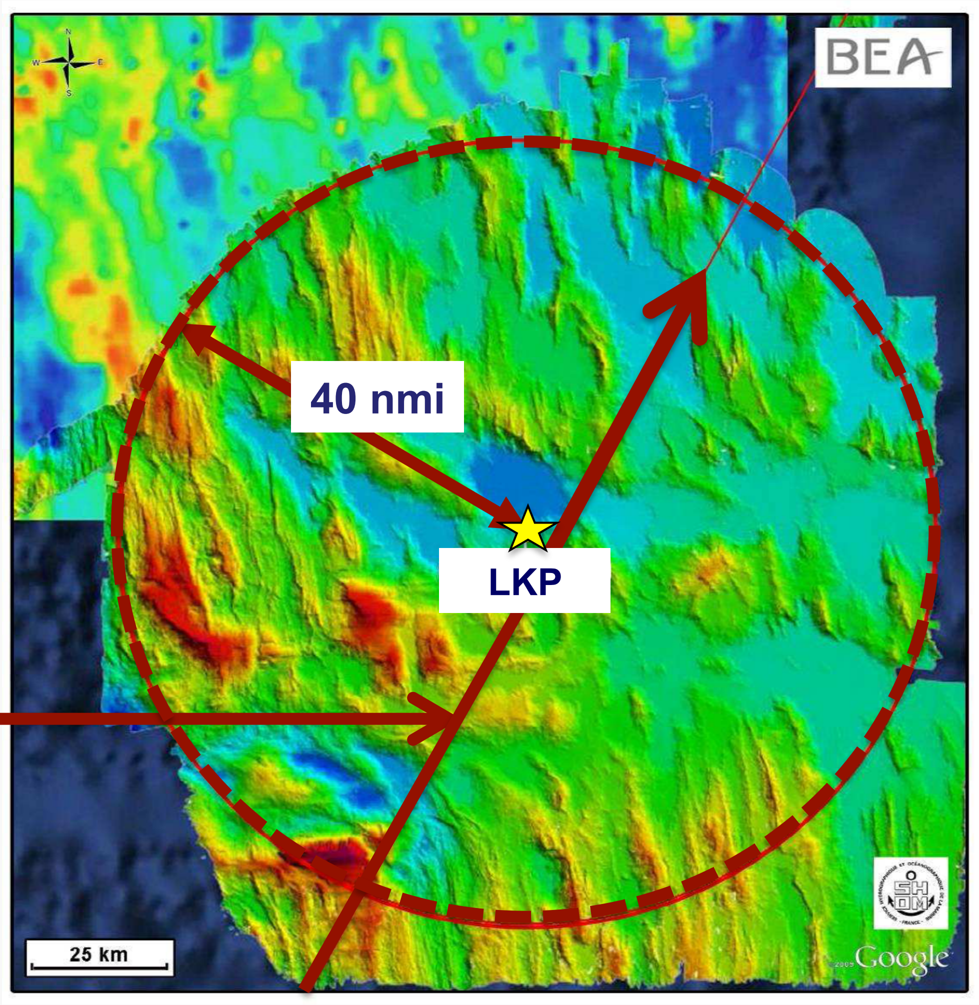


AF447 Disappears

Last Known Position (LKP) 2.98°N, 30.59°W

- In the early morning hours of 1 June 2009, Air France Flight 447, with 228 passengers and crew aboard, disappeared in convective weather over the South Atlantic
- The French Bureau of Enquiries and Analyses (BEA) took charge of the search.

Last Known Point and 40 nmi Search Zone



Intended Flight Path

Using the Airbus's satellite maintenance reporting system (ACARS), the BEA determined that the plane could have flown no farther than 40 nmi from the Last Known Point (LKP) before it crashed.

5,025 nmi^2 search area

Initial Searches

- Search operations for floating debris/survivors began the next day
- Five days later the first bodies and debris were found 38 nmi north of the LKP
- After the acoustic search for the “black box” beacons ended unsuccessfully, the BEA turned to Metron for assistance.

Tail Section

Galley

Analysis Process

Posterior after Phase 3 active sonar search

Posterior after Phase 2 active sonar search

Posterior after Phase 1 beacon search

Posterior after surface searches

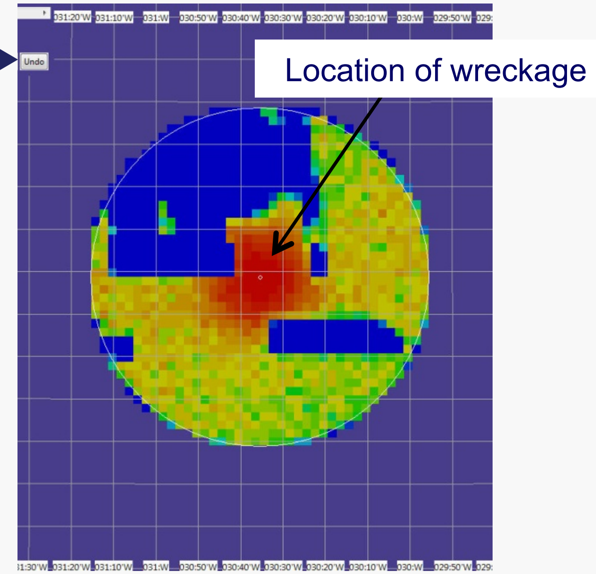
Prior Probability Distribution

35%
Uniform PDF over 40 nmi circle

35%
Circular normal PDF based on LOC accidents

Flight dynamics Scenarios

30%
PDF derived from reverse drift modeling

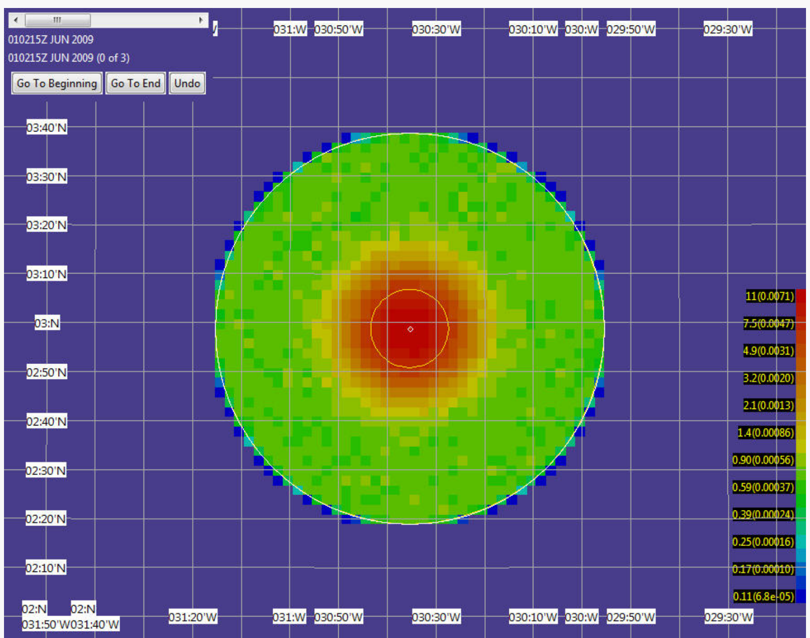


Final PDF leading to the wreckage discovery

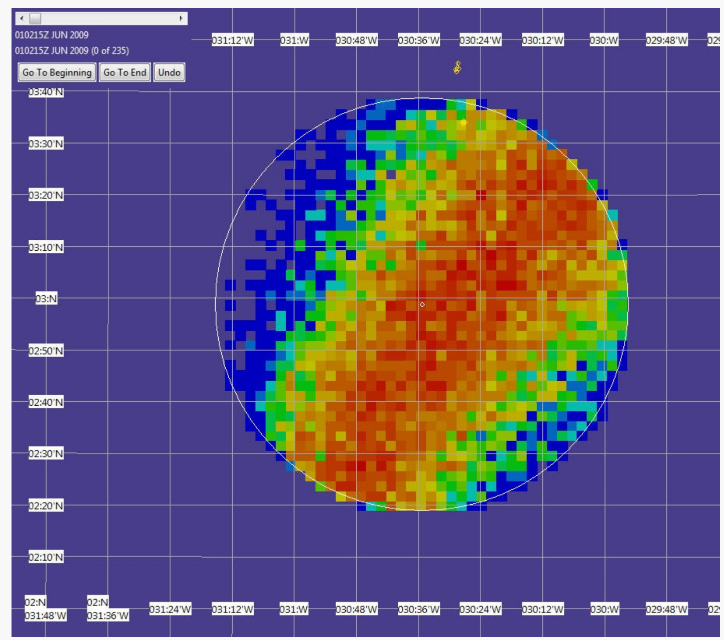
Flight Dynamics (FD) and Reverse Drift PDFs

The Prior PDF for impact was based on a weighted combination of three scenarios:

1. (35%) Uniform over 40 nmi circle about LKP
2. (35%) Distribution based on nine commercial accidents involving Loss of Control crashes – represented by circular normal with std dev of 8 nmi
3. (30%) SAROPS Reverse Drift simulation prior truncated at 40 nmi

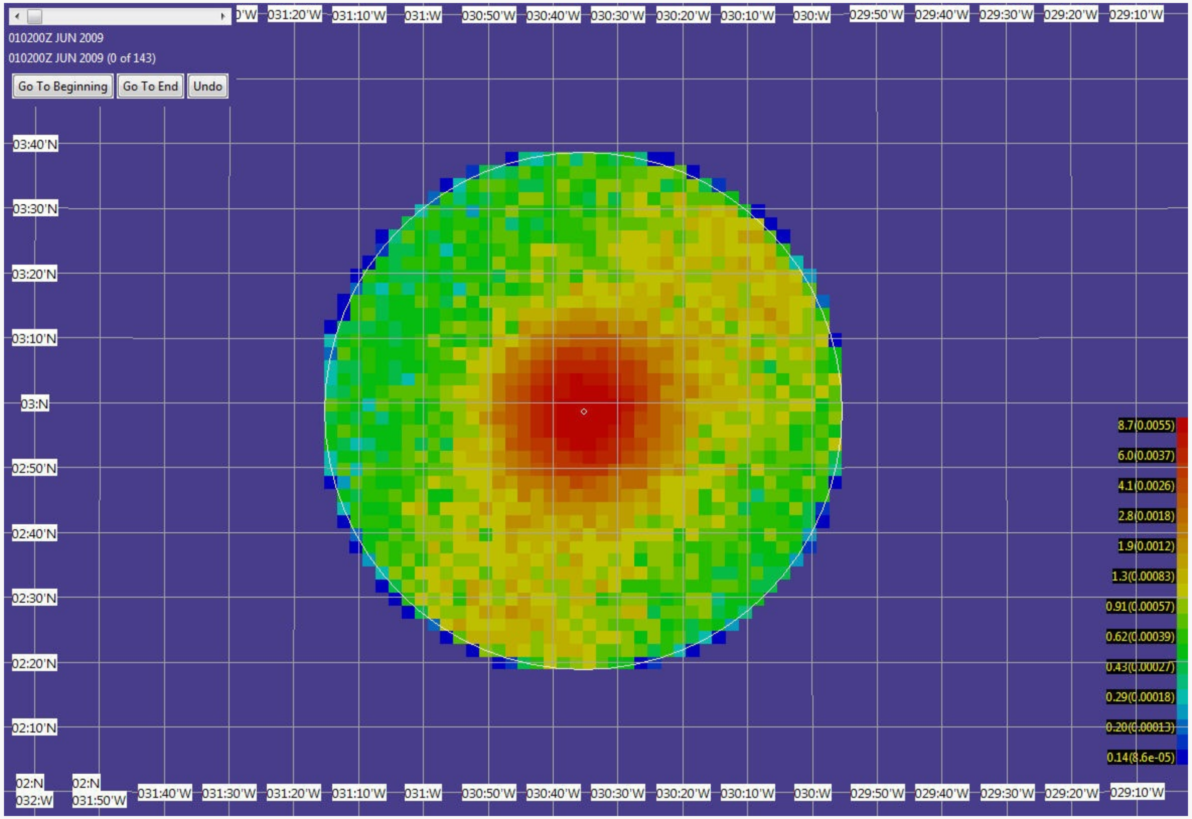


Flight Dynamics PDF



Reverse Drift PDF

Prior PDF



70% Flight Dynamics + 30% Reverse Drift

Analysis Process

Posterior after Phase 3 active sonar search

Posterior after Phase 2 active sonar search

Posterior after Phase 1 beacon search

Posterior after Surface Searches

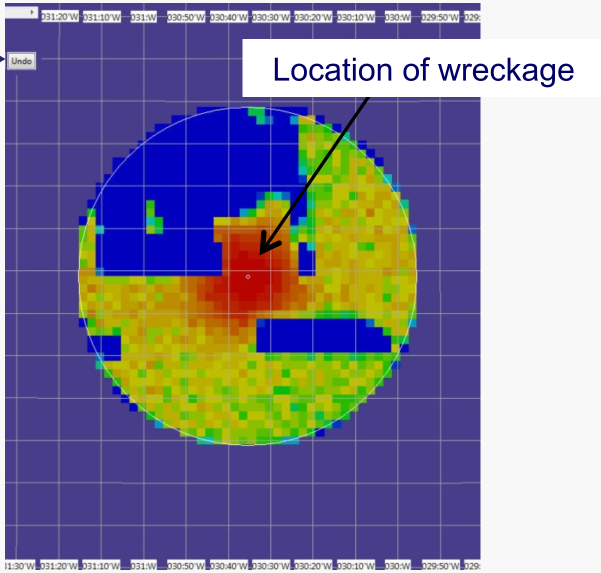
Prior Probability Distribution

Flight dynamics Scenarios:

- 35% Uniform PDF over 40 nmi circle
- 35% Circular normal PDF based on LOC accidents

Other scenario:

- 30% PDF derived from reverse drift modeling



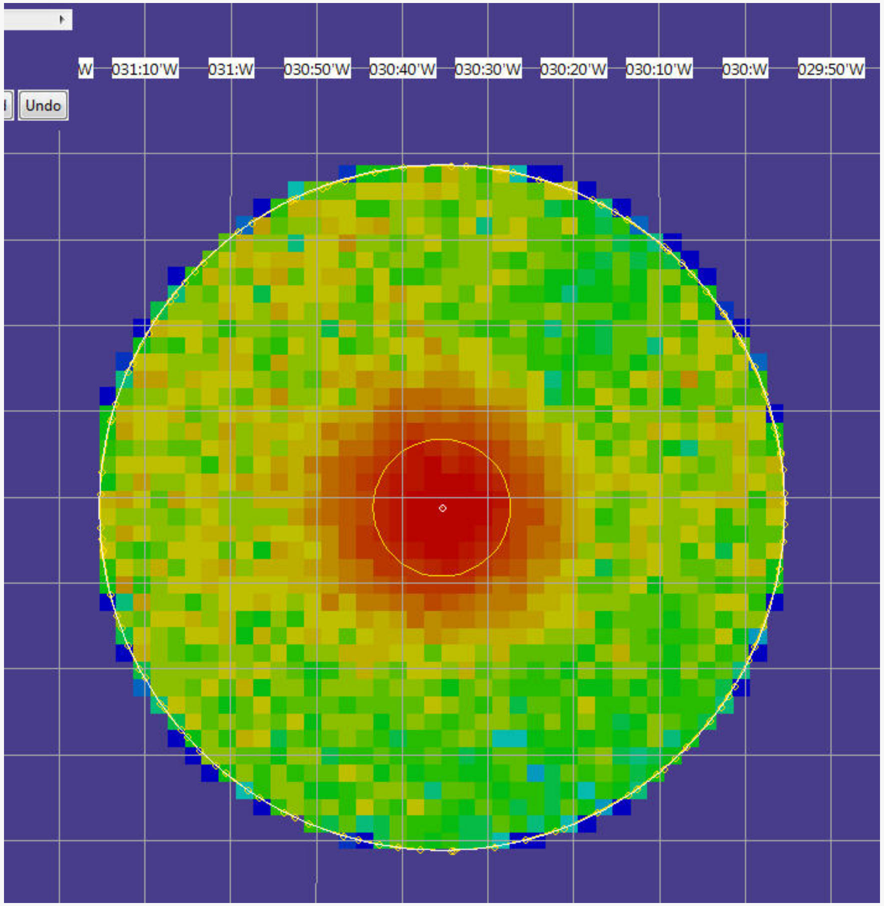
Final PDF leading to the wreckage discovery

Accounting for Failed Surface Search

(Negative information is still good information)

- Aircraft and ships searched the surface from 1 - 6 June before detecting floating debris
- Search paths for aircraft and ships put into SAROPS along with estimates of detection capability
- The Prior PDF was drifted forward in time using predicted currents.
  - For each path SAROPS computed probability the searches from 1 – 6 Jun fail to detect
  - Resulting failure probability used to weight each path.
- Final weighted paths pulled back to position at time of impact to form the Posterior PDF on impact position given failure of surface search

Surface Search Posterior PDF



Posterior after accounting for unsuccessful surface searches

Analysis Process

Posterior after Phase 3 active sonar search

Posterior after Phase 2 active sonar search

Posterior after Phase 1 beacon search

Posterior after surface searches

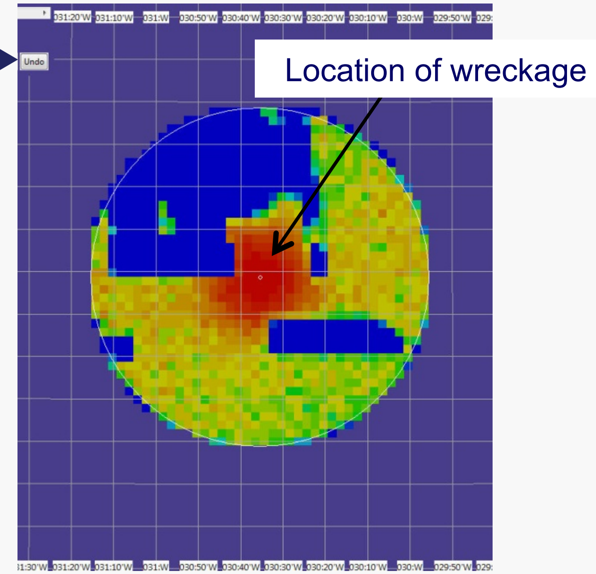
Prior Probability Distribution

35%
Uniform PDF over 40 nmi circle

35%
Circular normal PDF based on LOC accidents

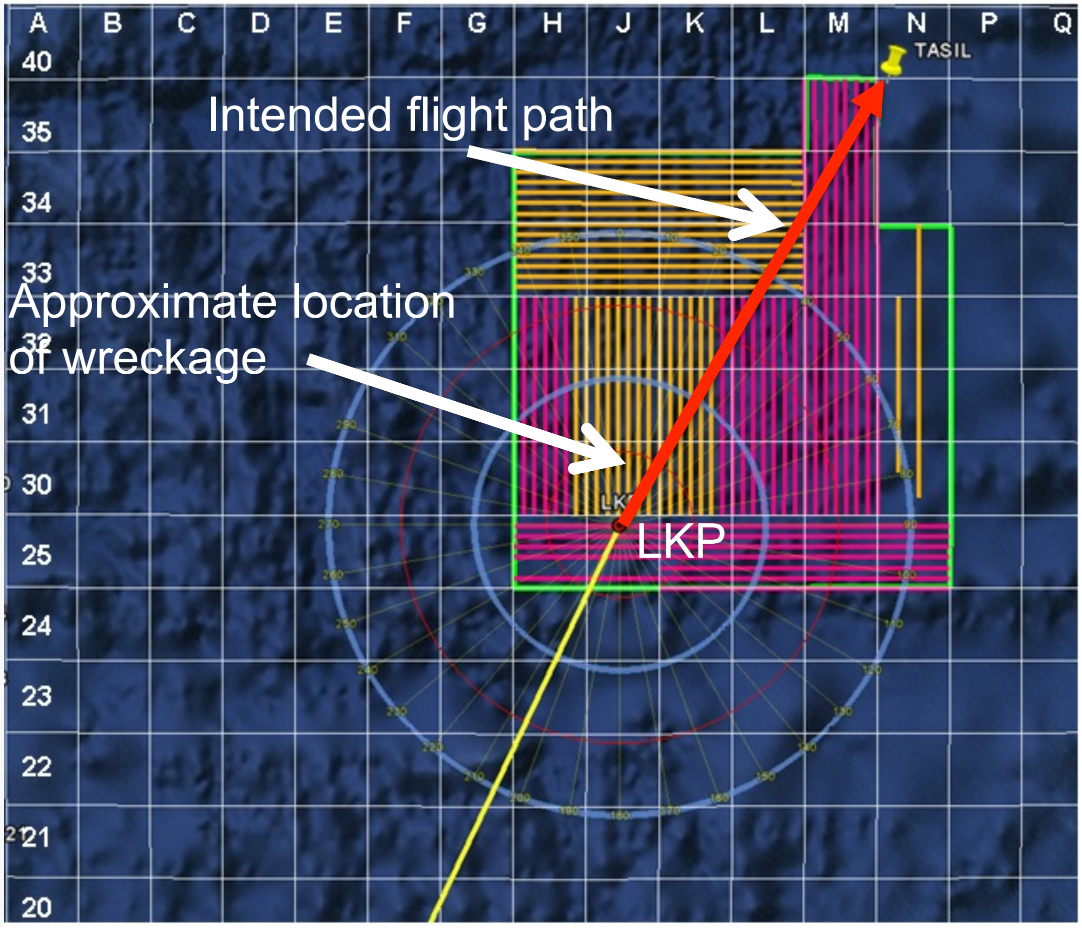
Flight dynamics Scenarios

30%
PDF derived from reverse drift modeling



Final PDF leading to the wreckage discovery

Phase 1: Towed Pinger Locator Search Paths



Assumed 0.8 probability of survival of each beacon.

- If survival is independent, then Pdet = 0.92 within 1730m lateral range
- If survival is dependent, Pdet = 0.72.

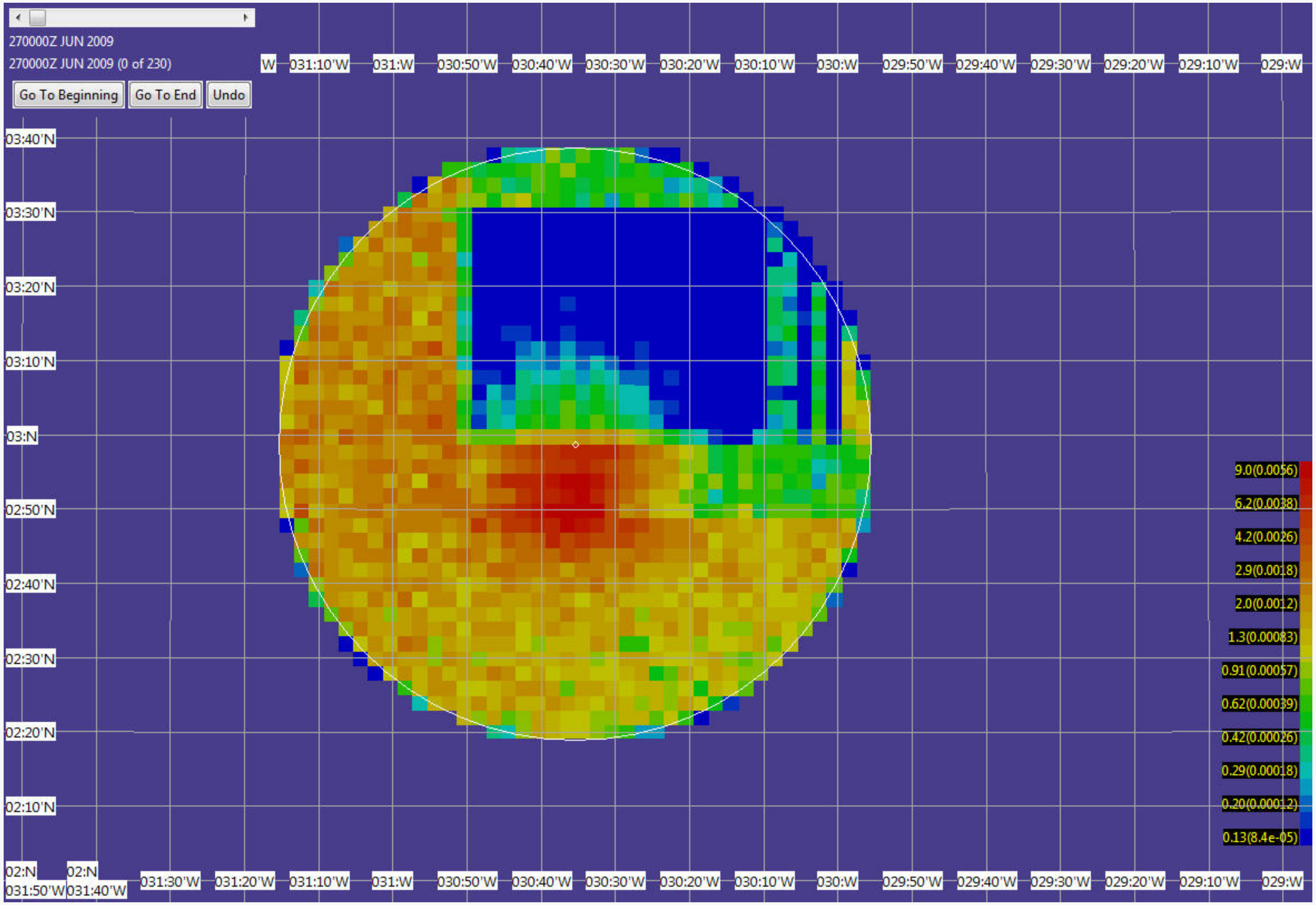
We used a weighted average for Pdet:

0.77 =(.25)(.92)+(.75)(.72)

Fairmount Glacier **(orange) and *Fairmount Expedition*** **(pink) Search Tracks.**

Blue circles are 20 NM and 40 NM circles about the LKP

Posterior after TPL Search



Analysis Process

Posterior after Phase 3 active sonar search

Posterior after Phase 2 active sonar search

Posterior after Phase 1 beacon search

Posterior after surface searches

Prior Probability Distribution

Flight dynamics Scenarios:

- 35%
  Uniform PDF over 40 nmi circle
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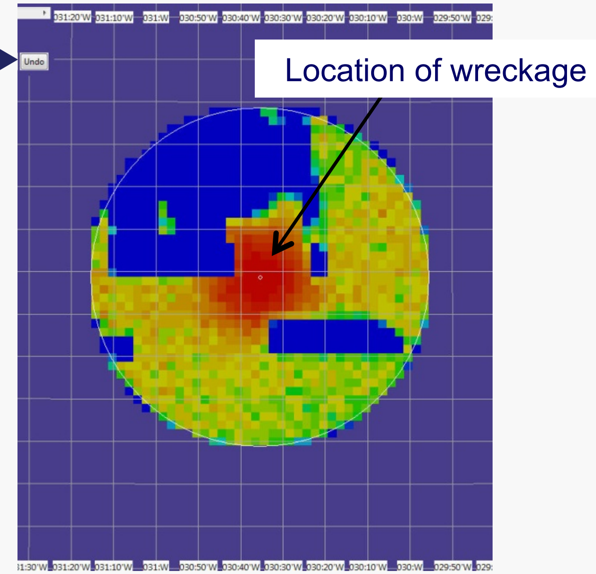
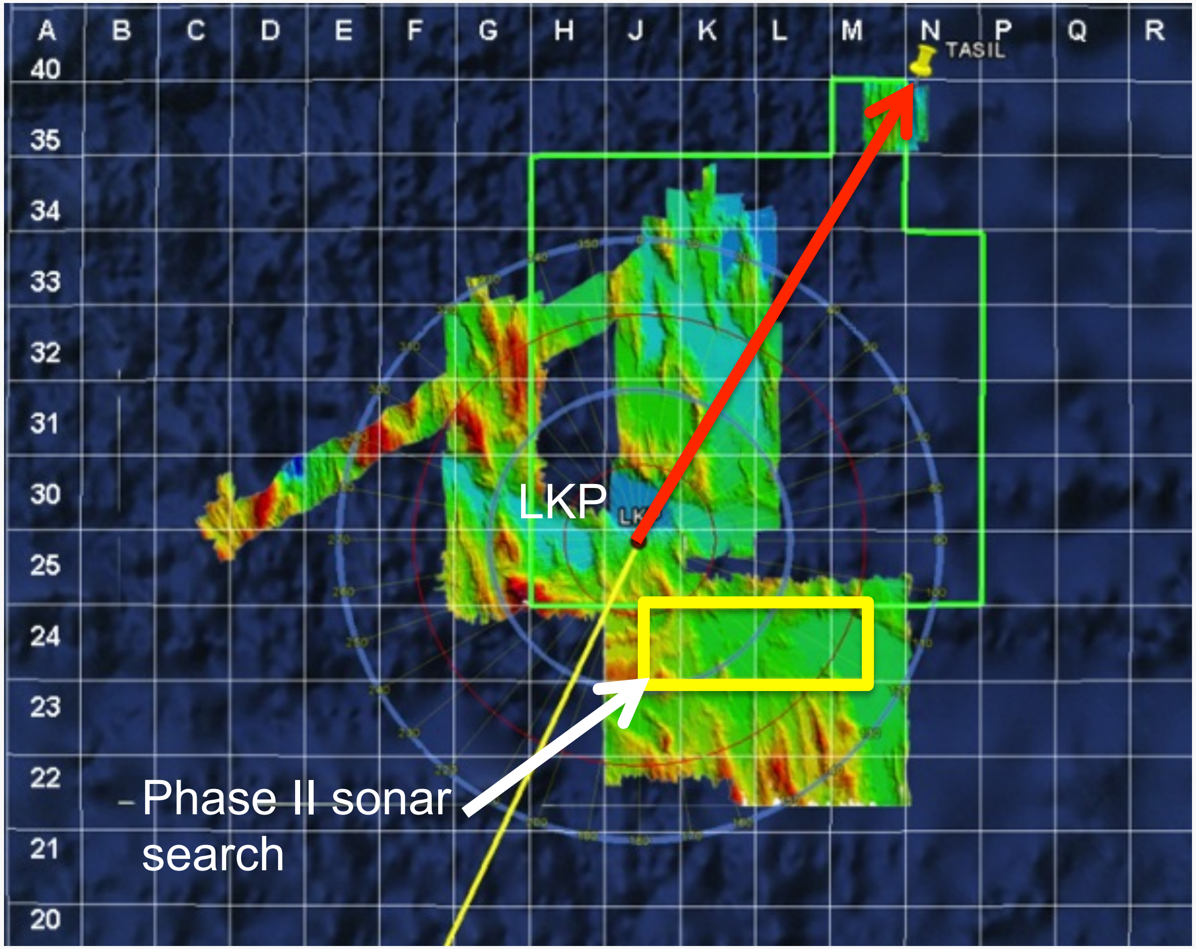
- 30%
  PDF derived from reverse drift modeling



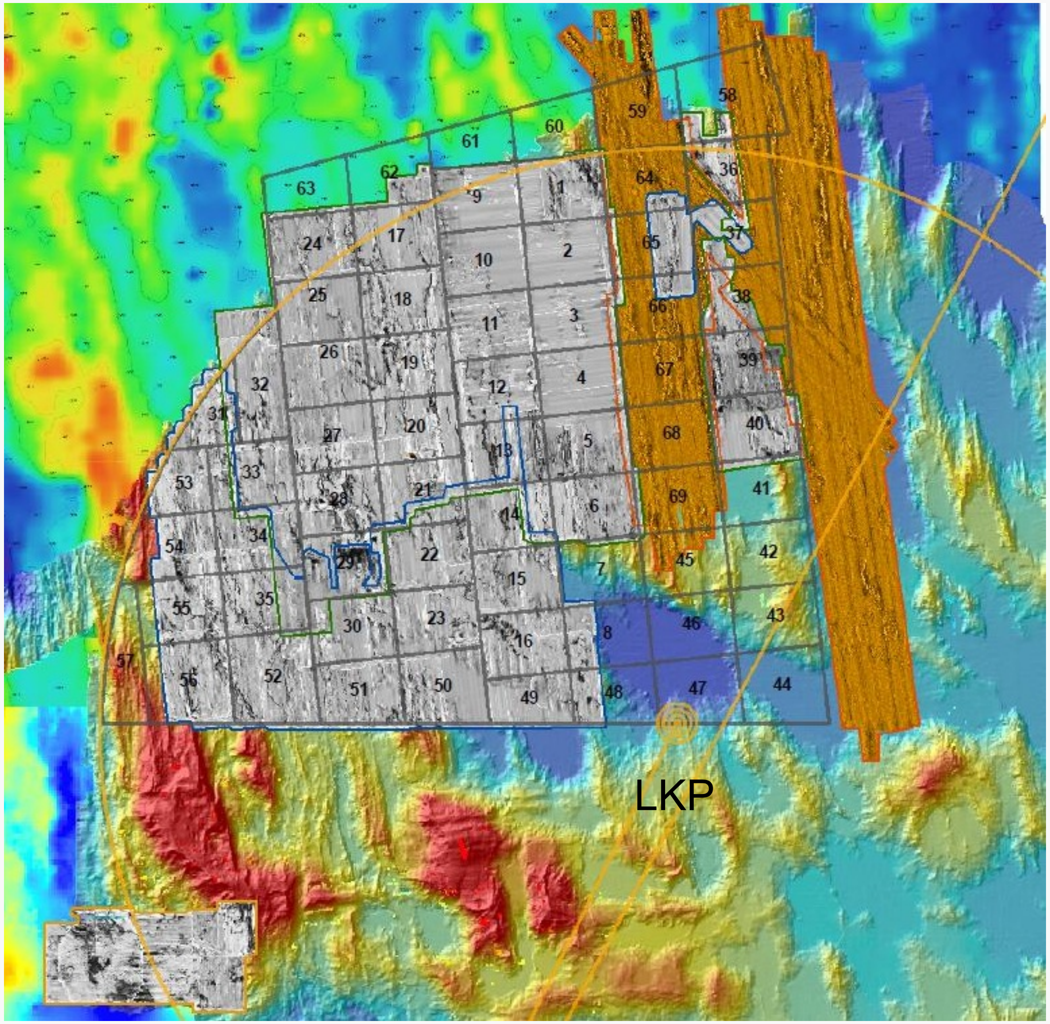
Final PDF leading to the wreckage discovery

Phase 2: Side-Scan Sonar



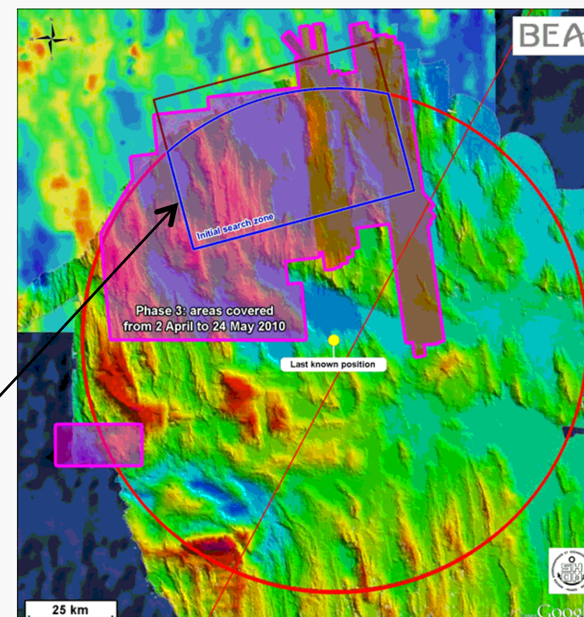
- In August 2009, the yellow outlined area was searched with a towed side scan sonar.
- Relatively flat bottom
- Pdet estimated at 0.9

Phase 3: REMUS and ORION Searches



- Woods Hole Oceanographic Institute (WHOI) deployed 3 REMUS 6000 AUVs which searched the grey area.
- US Navy/Phoenix International performed search in orange area using ORION towed side-looking sonar

Both searches rated highly effective, Pdet > 0.9

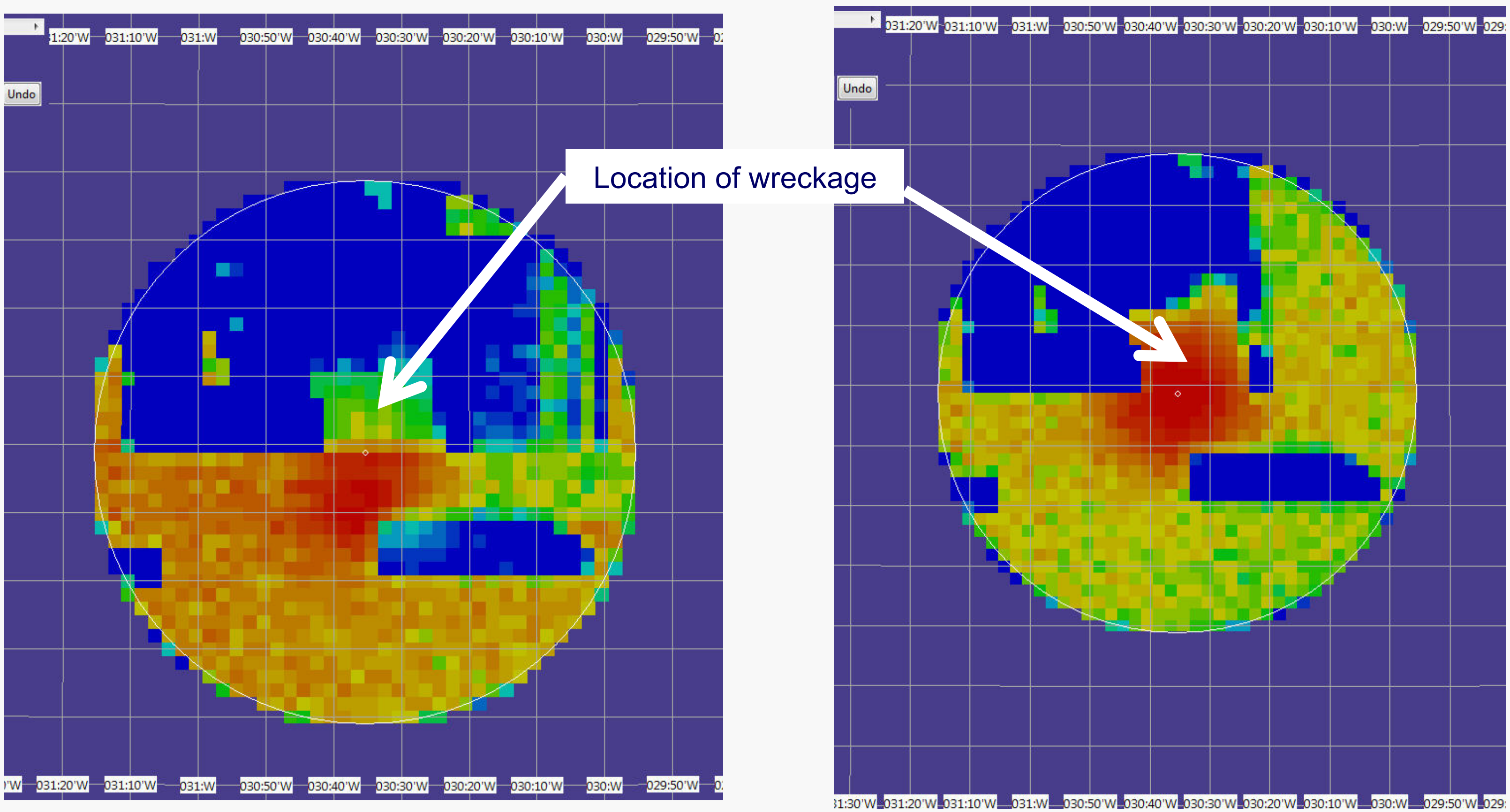


Search area based on reverse drift analysis by an international group of oceanographers

Posterior after Phases 2 and 3

In calculating the Posterior PDF at this point, we paused and considered the consequences of both beacons failing.

We opted to generate TWO PDFs…



Posterior assuming beacons worked

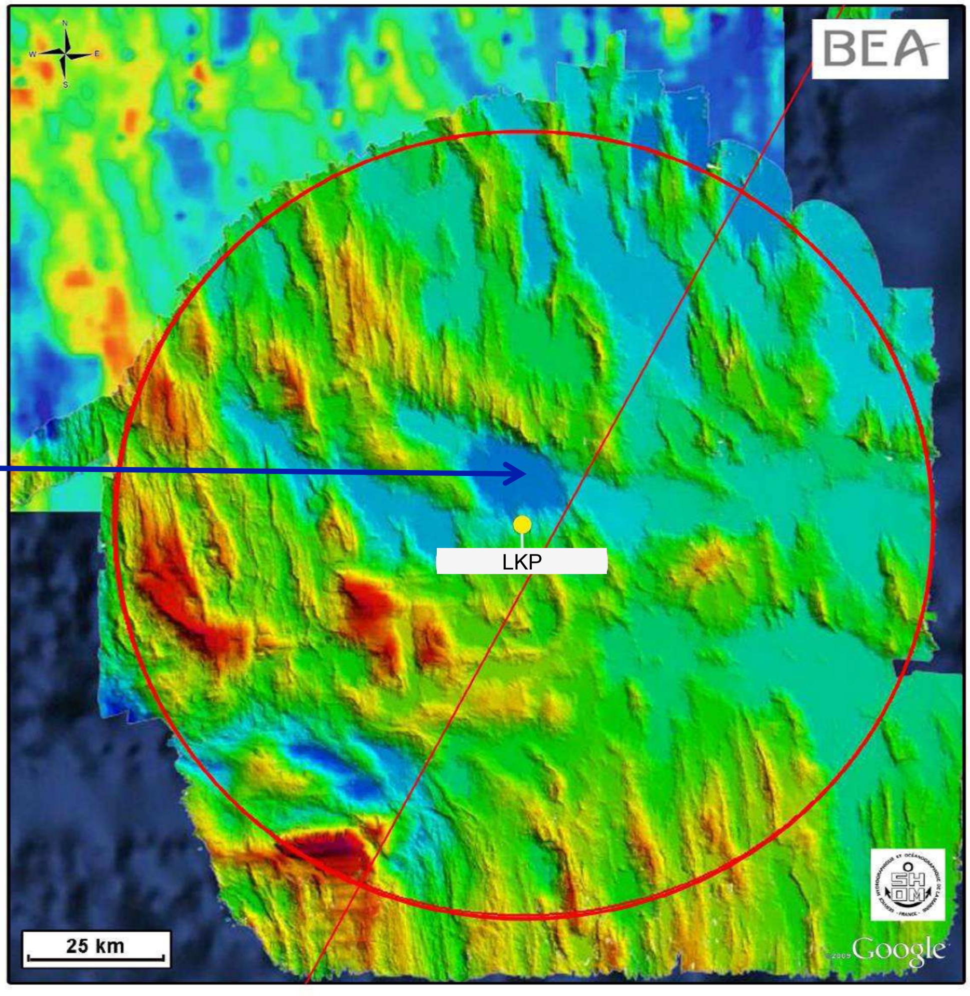
Posterior assuming beacons failed

Phase 4 Search Finds the Wreckage

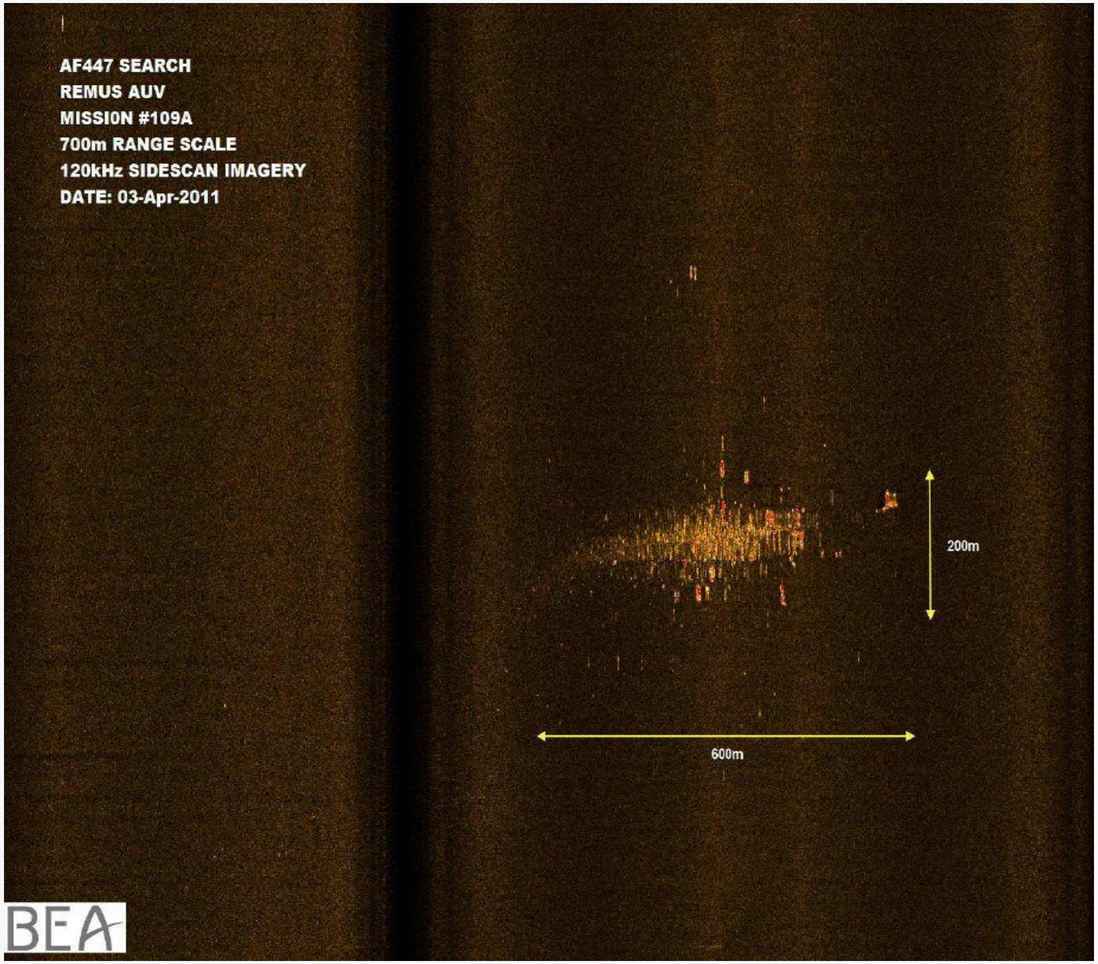
The BEA chose to use the second PDF and search near the LKP.

On 3 April 2011, the underwater wreckage was found.

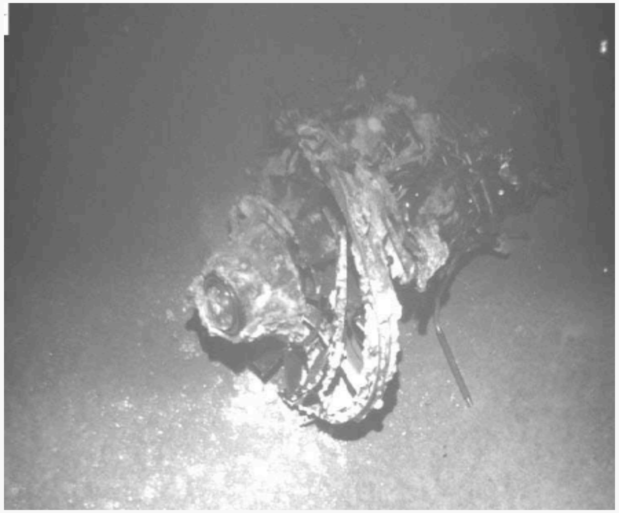
Approximate Location of Wreck



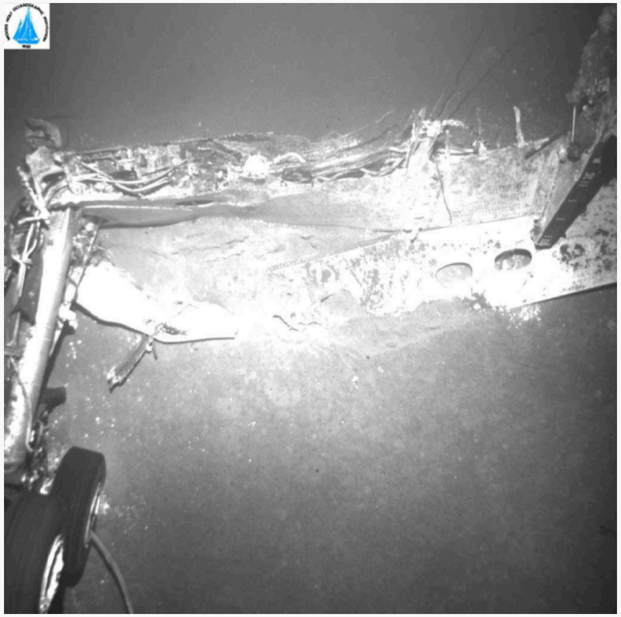
Wreckage



Side-Scan Sonar Image of Wreckage



Engine



Landing Gear

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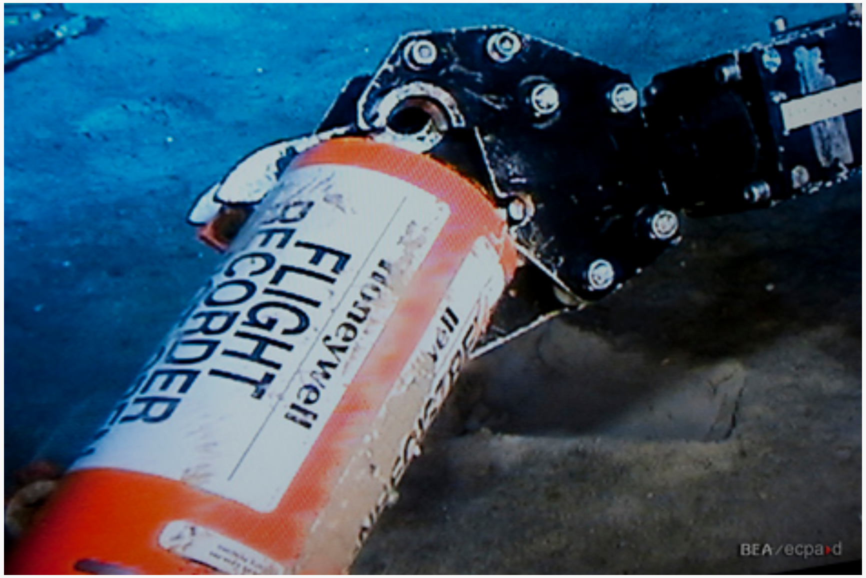
FDR and CVR with Beacons Recovered

The Flight Data Recorder (FDR) and Cockpit Voice Recorder (CVR) provided valuable information on the accident.



Flight Data Recorder (FDR) being recovered by a mechanical arm on the Remora 6000 Remotely Operated Vehicle

Cockpit Voice Recorder (CVR) capsule with the Underwater Locator Beacon (ULB) still attached

Result and Conclusions

- Result
  - Search began in high probability area of posterior distribution and found wreck within one week of effort
- Major Conclusion
  - The success of this effort provides a powerful illustration of the value of a methodical, Bayesian approach to search planning
- Minor Conclusions
  - The failure of the TPL search to detect the beacons led to a long and complicated search
  - Drift Group recommendations delayed success by a year
    - Used ad-hoc methods to determine search area

Bayesian Search Methods for MH 370

Comparison of MH 370 and AF 447

AF 447

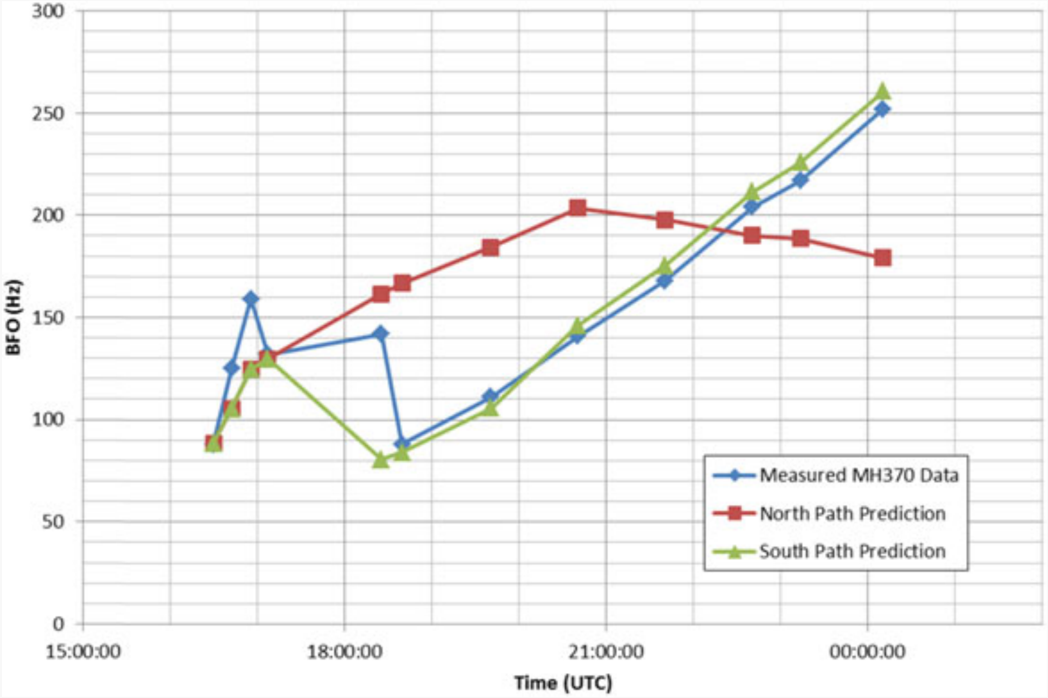
- Last known position GPS reported 2 min and 40 sec before crash
- Area of Uncertainty (40 nm circle): 17,240 km^2

MH 370

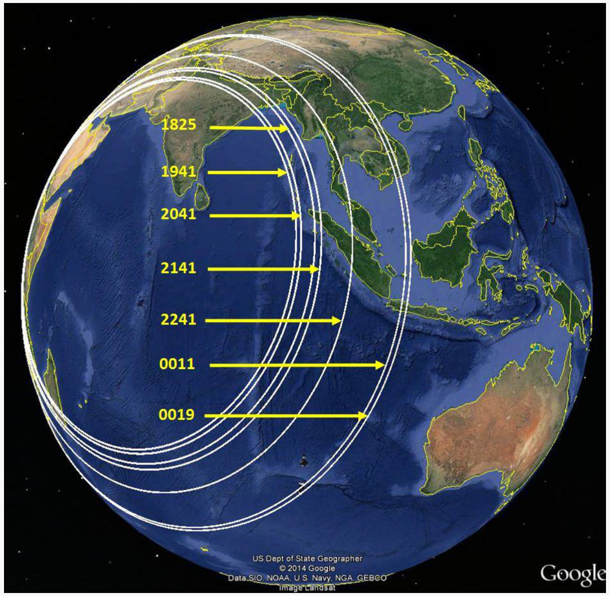
- Last radar fix 6 hrs before crash
  - Only satellite logon interrogations and requests after that
- Area of Uncertainty:
  - 60,000 km^2 – priority area (WV)
  - 240,000 km^2 – medium area (MI)
  - 1,120,000 km^2 – wide area (TX)

Developing the Prior

- LKP from radar at time 1822
- Series of Inmarsat satellite “handshakes” over the next 6 hours
  - Burst Time Offset (BTO) gives an estimate of aircraft range from the satellite
  - Burst Frequency Offset (BFO) identifies an arc along a given range ring.



BFO analysis identifying the southern flight path as the best fit to the observed data



BTO-derived Range Rings

How Would We Proceed?

- Step 1: Develop an aerodynamics-based prior distribution on flight paths
  - Flight paths could start at the last radar measurement and must provide position and (ground) velocity until time of crash.
- Step 2: Derive measurement (error) models for the BTO and BFO measurements.
  - Construct likelihood functions. A likelihood function computes the probability of receiving a measurement given a position and (ground) velocity of the aircraft at the time of the measurement
- Step 3: Compute the prior distribution on crash location. Specifically
  - Compute the likelihood of each path given the satellite measurements.
  - Multiply this times the aerodynamic prior probability
  - Normalize to compute posterior probability of each path
  - The weighted path endpoints yield a posterior probability distribution on crash location.

Aerodynamic Analysis of Flight Paths

- Simulate a large number of possible paths
  - Start paths at last radar fix generally heading Northwest and turn south by 1840 telephone call (BFO info)
  - Then follow procedure developed by Boeing[1]
    - Constant altitude,
    - Various autopilot modes (constant true track, constant true heading, constant true magnetic heading and great circle)
    - Speed and heading modeled by an Ornstein-Uhlenbeck process about mean
    - Model wind effects on ground speed
  - Terminated within 20 nmi of position at the last satellite handshake
  - Distribution weighted to have most locations less then 10 nmi.
- Result: A set of N paths giving 3-d position and velocity

$$\{(x_n(t), v_n(t)); n = 1, \ldots, N\} \text{ for } 1822 \le t \le 0019 \text{ plus final crash position}$$

  - Let p_n = probability of path n where $\sum_{n=1}^{N} p_n = 1$

1. MH370 – Definition of Search Areas, ATSB Transportation Safety Report, AE-2014-054, Final June 2014.

Likelihood Function

- Let g_1 and g_2 be the density functions for measurement errors of the BTO and BFO. (E.g., Normal with mean 0 and stand dev = 26 μs, 5 Hz)
  - Suppose we have BTO and BFO measurements Δt_i and Δf_i at time t_i
    - For each path n we calculate the predicted BTO and BFO, $\delta t(t_i, n)$ and $\delta f(t_i, n)$ given the position and velocity of the aircraft on path n at time t_i
  - The likelihood of these measurements given the aircraft follows the n th path is

$$l\left((\Delta t_i, \Delta f_i) \mid (x_n(t_i), v_n(t_i))\right) = g_1(\Delta t_i - \delta t(t_i, n))\, g_2(\Delta f_i - \delta f(t_i, n))$$

- The likelihood of obtaining the $I = 9$ measurements from logon requests and interrogations listed above given the aircraft follows the n th path is

$$L(n) = \prod_{i=1}^{9} g_1(\Delta t_i - \delta t(t_i, n))\, g_2(\Delta f_i - \delta f(t_i, n))$$

- The posterior probability on path n is

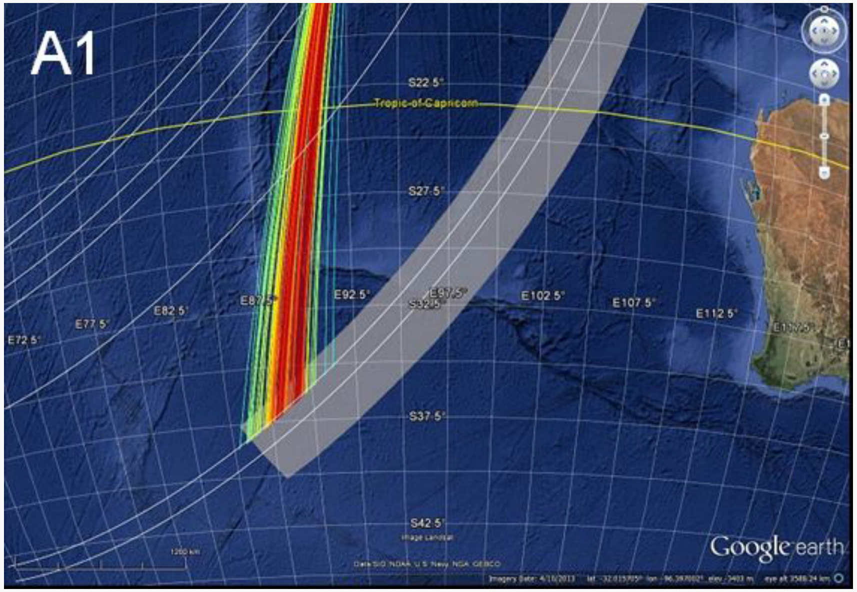
$$\tilde{p}_n = L(n) p_n \Big/ \sum_{m=1}^{N} L(m) p_m$$

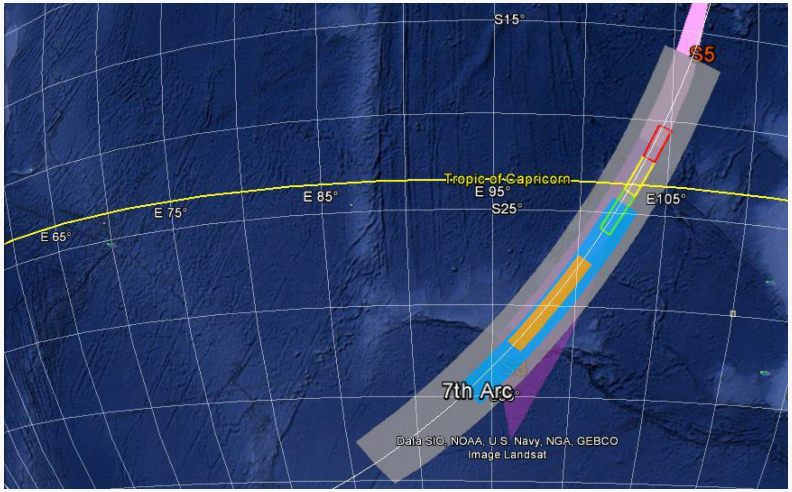
which yields the posterior probability on the crash point of path n.

Results

The result would look something like the following figure from the 8 Oct 2014 ATSB report,

- but with the paths extended to crash points to produce a probability distribution on crash location
- Since we don't have aircraft paths, hard to say how this would compare with search areas given by ATSB





Search areas:
Grey = wide   Blue = medium
Orange = priority

Results

- The probability distribution on the crash point would allow us to
  - Estimate probability of containment in search areas
  - Estimate probability of success as a function of search time (cost)
  - Recommend next area for search if initial search effort fails

Summary

- Benefits of Bayesian search planning
  - Provides a principled method of incorporating all information (objective and subjective) about the location of a search object to produce a probability distribution (PDF) on object location
  - Produces a PDF that provides the basis for efficient allocation of search effort
  - Allows incorporation of feedback from the search
  - Provides analytical estimates of the effort required to achieve a given level of probability of success and measures the effectiveness of search to date.
- Bayesian methodology has been applied successfully to a number of difficult searches involving lost aircraft and other objects
- The US Coast Guard routinely uses this methodology in its SAROPS system for planning searches for people and boats lost at sea.
  - You don't have to be a Bayesian search expert to use it.

References

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- MH370 – Flight Path Analysis Update, ATSB Transport Safety Report, AE-2014-054, 8 October 2014
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- Operations Analysis During the Underwater Search for *Scorpion,* by HR Richardson and LD Stone, *Naval Research Logistics*, 1971, 18:141-157
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